Recursive Least Squares with Forgetting for Online Estimation of Vehicle Mass and Road Grade: Theory and Experiments

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Abstract: Good estimates of vehicle mass and road grade are important in automation of heavy duty vehicle, vehicle following maneuvers or traditional powertrain control schemes. Recursive Least Square with multiple forgetting factors accounts for different rates of change for different parameters and thus, enables simultaneous estimation of the time-varying grade and the piece-wise constant mass. An ad-hoc modification of the update law for the gain in the RLS scheme is proposed and used in simulation and experiments. We demonstrate that the proposed scheme estimates mass within 5% of its actual value and tracks grade with good accuracy provided that inputs are persistently exciting. The experimental setups, signals, their source and their accuracy are discussed. Issues like lack of persistent excitations in certain parts of the run or difficulties of parameter tracking during gear shift are explained and suggestions to bypass these problems are made.

1 Introduction

In vehicle control, many control decisions can be improved if the unknown parameters of the vehicle model can be estimated. Weight of the vehicle, coefficient of rolling resistance, and drag coefficient are examples of unknown parameters. Road grade is a major source of external loading for heavy vehicle longitudinal dynamics and is normally unknown. Both mass and grade are found to be critical in brake-by-wire and vehicle-following maneuvers. The mass of a heavy duty vehicle can vary as much as 400% depending on the load it carries. Mild grades for passenger vehicles, are serious loadings for heavy vehicles. An anti-lock brake controller relies on an estimate of mass and road grade for calculating vehicle's cruise speed which is necessary for estimation of wheel slip. In longitudinal control of

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platoons of mixed vehicles, knowledge of the participating vehicle mass and road grade is necessary for avoiding issuing infeasible acceleration and braking commands [4]. Moreover, mass estimation is essential to the engine control unit (ECU) for reduced emission, and to transmission control for reduced gear hunting. The closed loop experiments performed by Yanakiev et al. [31] indicate that the longitudinal controllers with fixed gains have limited capability in handling large parameter variations of an HDV. Therefore it is necessary to use an adaptive control approach with an implicit or explicit online estimation scheme for estimation of unknown vehicle parameters.

Examples of adaptive controllers for vehicle control applications can be found in the work by Liubakka et al. [19], Ioannou et al. [15], and Oda et al [21]. Yanakiev et al. [32, 33] have developed an adaptive controller for longitudinal control of an HDV using direct adaptation of PIQ controller gains. Recently, Druzhinina et al. [11] have developed an adaptive control scheme for longitudinal control of HDV's. Within this scheme they provided simultaneous mass and road grade estimation. They demonstrated convergence in estimates for constant mass and piecewise constant grade. This method is an indirect estimation method since estimation is achieved in closed-loop and as a by-product of a control scheme.

As HDV automation is increasing, there are more controllers that could benefit from on-line estimation of the vehicle mass and road grade. Moreover many times estimates independent of a controller are required. In other words a direct estimation scheme is more appealing. The proposed schemes for direct estimation of vehicle parameters, particularly mass and grade can in general be classified in two categories: sensor-based and model-based methods. In sensor based methods some type of extra sensor is used on the vehicle to facilitate estimation of one or more parameters. Model-based schemes use a model of the vehicle and data like engine torque, vehicle speed, engine speed and gear ratio which are available through the CanBus to estimate the unknown parameters. Since longitudinal dynamics of the vehicle depends on both mass and grade, knowing one will facilitate estimation of the other. Therefore some suggest estimating the grade which is in general time varying with some type of sensor and then estimating the mass with conventional parameter-adaptive algorithms [15, 30]. Bae et al. [5] use GPS readings to obtain road elevation and calculate the grade using the measured elevations. With the grade known, they estimate the mass with a simple least square method based on the longitudinal dynamics equation. In [22] using an on-board accelerometer is proposed for grade estimation. The mass is then estimated based on a good approximations of the grade.

A model-based method can provide a cheap alternative in estimation or it can be used along with a sensor-based scheme to provide some redundancy. One approach [13] which has been patented and has been used in industry is estimation of mass based on the velocity drop during a gearshift. The idea is that since the gearshift period is short, the road load can be rendered constant. The change in velocity before and during gearshift can be used to calculate an estimate for the mass based on the longitudinal dynamics equation. However based on a fair amount of trial, we observed that the velocity drop is normally minor during a gearshift and this limits the accuracy of the method due to the small signal-to-noise ratio. Besides, this approach does not address estimation of the grade.

In the rest of this paper a direct approach for simultaneous estimation of mass and time-varying grade is pursued. We first formulate vehicle longitudinal dynamics and explain experimental setups and validation of longitudinal model. We then investigate implementation of a recursive least square (RLS) method for simultaneous online mass and grade estimation. We briefly discuss the recursive least square scheme for time varying parameters and review some key papers that address the subject. The difficulty of the popular RLS with single forgetting is discussed next. For estimation of multiple parameters which vary with different rates, RLS with vector-type forgetting is previously proposed in a few papers. We analyze this approach and propose an ad-hoc modification of the update law for the gain in the RLS scheme. Although, we could not prove the algorithm convergence, nor define a region of convergence for the algorithm, we demonstrate, with both simulated and test data, that incorporating two distinct forgetting factors is effective in resolving the difficulties in estimating mass and time-varying grade. The experimental setup and particular issues with experimental data are also discussed.

2 Vehicle Longitudinal Dynamics

Our estimation approach is a model based approach. That is, we rely on a physical model of vehicle's longitudinal dynamics and use this model and the data that is recorded from the vehicle's CanBus for estimating mass and grade and possibly other unknown parameters which affect vehicle's longitudinal motion. Therefore we first formulate the vehicle longitudinal dynamics equation.

A vehicle's acceleration is a result of combination of engine and braking torques and the road loads on the vehicle. When the torque convertor and the driveline are fully engaged we can assume that all the torque from the engine is passed to the wheel. Further assuming that there is no wheel slip, which is a good assumption for most part of the motion, the longitudinal dynamics can be presented in the following simple form:

$$M\dot{v} = \frac{T_e - J_e\dot{\omega}}{r_g} - F_{fb} - F_{aero} - F_{grade} \tag{1}$$

In this equation M is the total mass of the vehicle, v is the forward speed and ω is rotational engine speed. T_e is the engine torque at the flywheel. If engine is in fuelling mode the torque is positive and if it is in the compression braking mode the torque is negative. If the transmission and the torque convertor are fully engaged then most of the torque is passed to the wheels as assumed in the above equation. To model the possible torque losses, engine torque can be scaled down by a coefficient of efficiency. J_e is the powertrain inertia and therefore the term $J_e\dot{\omega}$ represents the portion of torque spent on rotating the powertrain. r_g is the wheel radius divided by total gear ratio:

$$r_g = \frac{r_w}{g_d g_f}$$

where r_w is the wheel radius, g_d is the gear ratio and g_f is the final drive ratio. F_{fb} is the generated friction brake (service brake) force at the wheels. The force due to aerodynamic resistance is given by

$$F_{aero} = \frac{1}{2} C_d \rho A v^2$$

where C_d is the drag coefficient, ρ is air density and A is frontal area of the vehicle. F_{grade} describes the combined force due to road grade (β) and the rolling resistance of the road (μ). It is given by

$$F_{grade} = Mg(\mu \cos\beta + \sin\beta),$$

where g is the gravity constant. Here $\beta = 0$ corresponds to no inclination, $\beta > 0$ corresponds to uphill grade and $\beta < 0$ represents downhill. Eq. (1) is valid when the wheels do not have considerable slip.

We are interested in using this equation along with the data obtained from vehicle's CanBus for online estimation of mass and grade. In section 3 signal measurement and identification of model parameters are explained.

Equation (1) can be rearranged so that mass and grade are separated into two terms:

$$\dot{v} = (\frac{T_e - J_e\dot{\omega}}{r_g} - F_{fb} - F_{aero})\frac{1}{M} - \frac{g}{\cos(\beta_\mu)}\sin(\beta + \beta_\mu) \tag{2}$$

where $\tan(\beta_\mu) = \mu$. We can rewrite Eq. (2) in the following linear parametric form,

$$y = \phi^T\theta,\ \phi = [\phi_1, \phi_2]^T\ \theta = [\theta_1, \theta_2]^T \tag{3}$$

Where

$$\theta = [\theta_1, \theta_2]^T = [\frac{1}{M},\ \sin(\beta + \beta_\mu)]^T$$

are the unknown parameters of the model, which we try to estimate and

$$y = \dot{v},\ \phi_1 = \frac{T_e - J_e\dot{\omega}}{r_g} - F_{fb} - F_{aero},\ \phi_2 = -\frac{g}{\cos(\beta_\mu)}$$

can be calculated based on measured signals and known variables.

Had the parameters θ_1 and θ_2 been constant, a simple recursive algorithm, like recursive least squares, could have been used for estimation. However while θ_1 depends only on mass and is constant, the parameter θ_2 is in general time-varying. Tracking time-varying parameters needs provisions that we directly address later in this paper.

3 Experimental Setup

We planned experiments on a Freightliner truck owned by California PATH [1]. The signals are measured through different interfaces. The CanBus, which is available on the vehicle, is responsible for communication between the engine and powertrain controllers. Many of the signals are obtained by accessing the CanBus. The signals are transferred under certain standards set by SAE [2]. Currently the J1939 [1] and its extensions like J1939-71[2] are standard for heavy duty vehicles. Older equivalents are SAE J1587 for powertrain control applications. Other sources are EBS, GPS and customized sensors installed by PATH staff. The EBS is the electronic brake control system and measures signals like wheel speed. A GPS antenna is available on the PATH truck that provides, longitude, altitude and latitude coordinates as well as the truck's cruise speed. A few sensors had been installed on the truck including accelerometers in x, y and z directions, tilt sensors,and pressure transducers for measuring brake pressure at the wheels.

The real time QNX operating system was used for data acquisition. The system was wired to the Canbus and other sensors and data was sampled at 50 Hz. A computer specialist monitored the flow of data and logged the instructions and actions by the driver and other researchers in a text file that was available to us after the test. The whole test was carried out open-loop except for some periods when cruise control was activated. Each run concentrated on gathering data required for identification of one or more components such as service brakes, compression brake, gear scheduling, etc. For successful identification we made sure that the dynamics is sufficiently rich, many times by asking the driver to pulse the commands like throttle and braking. To generate different loading scenarios, the loading of the trailer was decreased gradually from full to empty in stages during the test. At each stage the total mass of the truck was known. Abundant amount of data in distinct driving scenarios was obtained during two days of test. In the next sections we explain how the data was used for system identification and parameter estimation.

3.1 Measured Signals

Numerous signals are recorded during the experiments, based on different sensors, each with certain degree of accuracy, and different levels of noise. The update rates and sampling rates for the signals might also vary from one to the other based on the sensors and the port they are read from. In this section we discuss the source and accuracy of data. Then we proceed to estimate the parameters based on this measured data.

Velocity is available from J1939 as well as the EBS sensors which measure the wheel speed. GPS also provides an accurate measure of the velocity. Engine speed is known from J1939 with good accuracy. Engine torque, compression brake and transmission retarder torques are available through the J1939 port. These engine and compression torques are

[1] Partners for Advanced Transit and Highways
[2] Society of Automotive Engineers

calculated based on static engine maps and do not reflect the very fast dynamics of the engine. However they are fast enough for our purpose. Pressure transducers are installed to measure the brake pressure at the wheels. Determining the actual force developed by service brakes will depend on a model that translates the pressure into a torque. At this stage we do not have such a model and therefore in our analysis we will dismiss portions of data in which service brakes were activated. The transmission status is available form J1939. That determines if the driveline is engaged and whether the torque convertor is locked or if a shift is in process. The driveline is always flagged engaged when not in neutral. The torque convertor was shown locked whenever the vehicle was in the third or a higher gear. Shift in process denotes the period of a gear shift when the transmission controller is in effect. The gear number could not be accessed through J1939 at the time of the test. So the J1587 port was used to get the gear numbers. Each gear ratio and the final drive ratio were available from the transmission manufacturer and were also verified using data available from J1939.

The signals recorded from the accelerometers were noisy and therefore we decided not to use these signals for obtaining accelerations. Also the signals recorded from tilt sensors had a small signal to noise ratio and therefore we could not investigate possibility of using tilt sensors for measuring the road grade. The actual road grade was extracted from the profile plans of the road.

3.2 Road Grade

The road tests were carried out on a part of the HOV lane of Interstate 15 north of San Diego. Within the two days of test, various driving cycles were completed in a number of round trips on a twelve kilometer stretch of highway. The test route included some overpasses with steep grades. This grade was later determined using the road plans and served as a comparison with the estimated grade. Although the GPS elevation signal was available in the test-run, the information was often noisy or corrupted as shown in the upper subplot of figure 1. The most accurate source for the road grade is the as-built plan available for roads and highways. Therefore we obtained the profile plans of the experimental track from Caltrans [3]. We then carefully digitized the plans and determined the grade based on the elevations. Figure 1 shows the digitized elevation and grade. Note that the grade is either constant or varies linearly with distance. That is a natural result of highway design where the transition between slopes are parabolic. We used the information from GPS to determine the starting point of each test run on the digitized elevation map.

3.3 Determining Unknown Parameters

In the vehicle longitudinal dynamics equation 2, wheel radius, r_w, driveline inertia, J_e, drag coefficient, C_d, and coefficient of rolling resistance, μ, were unknown. Extra care

[3]California Department of Transportation

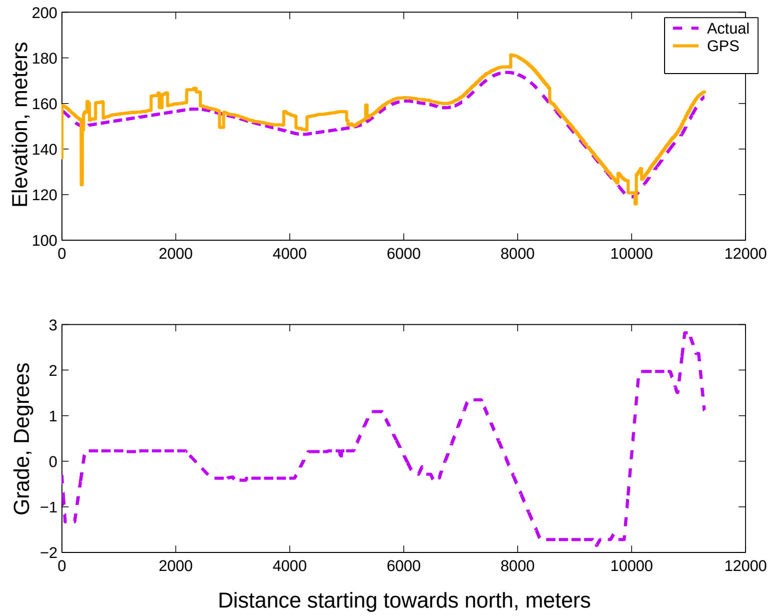

Figure 1: Digitized road elevation and grade.

was taken to obtain an accurate value for tire rolling radius, r_w, since other parameters are sensitive to this value. It was calculated based on the gear ratios available from the transmission manufacturer, engine speed and vehicle velocity which are available from J1939. This value was also verified by tape measurement of the drive wheel radius on site. r_g could be calculated based on this tire radius and gear ratios. J_e was not available for the experimental truck. We used a value available from another truck. However sensitivity to this parameter is not very high and deviations from this nominal value can be tolerated.

A range for values of drag coefficient and coefficient of rolling resistance for different vehicles is available in handbooks of vehicle dynamics (e.g. [29]). To select the values that fit our available data we used the vehicle longitudinal dynamics equation (1) and tuned the parameters of the model to make the outcomes roughly match the experimental data. The model used the engine or the retarder torque, the road grade and the selected gear that were recorded during the test and based on these inputs the accelerations were calculated. The accelerations were compared to the accelerations obtained from the test data. The drag coefficient and rolling resistance were tuned in the feasible range so that calculated and actual accelerations roughly matched each other. We found coefficient of rolling resistance of 0.006 and drag coefficient of 0.7 suitable candidates that result in good match between experiments and simulation. Figure 2 shows a typical test run with good match between test data and simulation results for most part of the trip. During gear changes experiments and simulation results do not have a good match. This is due to the fact that the gear shift dynamics is not considered in the longitudinal dynamics model. In the model we have assumed that velocity and engine speed are always proportionally related and

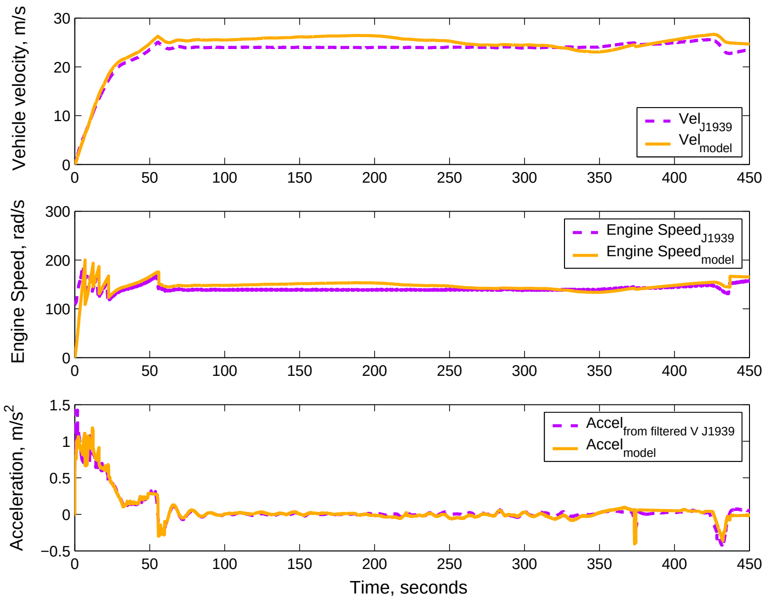

Figure 2: Comparison of the model and real longitudinal dynamics.

that transmission is always engaged. These assumptions only result in local mismatch between model and experiments and in general the model represents the longitudinal dynamics adequately well.

Having identified the model of vehicle longitudinal dynamics, we continue with the theory of RLS estimation and the proposed algorithm.

4 Recursive Least Square Estimation

In least square estimation unknown parameters of a linear model are chosen in such a way that the sum of the squares of the difference between the actually observed and the computed values, is a minimum [3]. For a linear system (e.g. model shown in (3)) this translates into finding the parameter(s) that minimizes the following "loss-function",

$$V(\hat{\theta}, n) = \frac{1}{2} \sum_{i=1}^{n} \Big(y(i) - \phi^T(i)\hat{\theta} \Big)^2 \tag{4}$$

Solving for the minimizing parameters we get the closed form solution as follows:

$$\hat{\theta} = \left(\sum_{i=1}^{n} \phi(i)\phi^T(i) \right)^{-1} \left(\sum_{i=1}^{n} \phi(i)y(i) \right) \tag{5}$$

Most of the time we are interested in real-time parameter estimation. Therefore it is computationally more efficient if we update the estimates in (5) recursively as new data becomes available online. The recursive form is given by:

$$\hat{\theta}(k) = \hat{\theta}(k-1) + L(k)\left(y(k) - \phi^T(k)\hat{\theta}(k-1)\right) \tag{6}$$

where

$$L(k) = P(k)\phi(k) = P(k-1)\phi(k)\left(1 + \phi^T(k)P(k-1)\phi(k)\right)^{-1} \tag{7}$$

and

$$P(k) = \left(I - L(k)\phi^T(k)\right)P(k-1) \tag{8}$$

$P(k)$ is normally referred to as the covariance matrix. More detailed derivation can be found in books on parameter estimation such as [3]. For convergence proof see for example the book by Johnson [16].

Eq.(6) updates the estimates at each step based on the error between the model output and the predicted output. The structure is similar to most recursive estimation schemes. In general most have similar parameter update structure and the only difference is the update gain $L(k)$. The scheme can be viewed as a filter that averages the data to come up with optimal estimates. Averaging is a good strategy if parameters of the model are constant in nature. However many times the parameters that we are estimating are time-varying and we are interested to keep track of the variations. In the next section the generalized RLS for time-varying parameters is discussed.

4.1 Recursive Least Square Estimation with Forgetting

If the values of the parameters of a system change abruptly, periodic resetting of the estimation scheme can potentially capture the new values of the parameters. However if the parameters vary continuously but slowly a different heuristic but effective approach is popular. That is the concept of forgetting in which older data is gradually discarded in favor of more recent information. In least square method, forgetting can be viewed as giving less weight to older data and more weight to recent data. The “loss-function” is then defined as follows:

$$V(\hat{\theta}, k) = \frac{1}{2}\sum_{i=1}^{k} \lambda^{k-i}\left(y(i) - \phi^T(i)\hat{\theta}(k)\right)^2 \tag{9}$$

where λ is called the forgetting factor and $0 < \lambda \leqslant 1$. It operates as a weight which diminishes for the more remote data. The scheme is known as least-square with exponential forgetting and θ can be calculated recursively using the same update equation (6) but with $L(k)$ and $P(k)$ derived as follows:

$$L(k) = P(k-1)\phi(k)\left(\lambda + \phi^T(k)P(k-1)\phi(k)\right)^{-1} \tag{10}$$

and

$$P(k) = \left(I - L(k)\phi^T(k)\right)P(k-1)\frac{1}{\lambda}. \tag{11}$$

The main difference with the classical least square method is how the covariance matrix $P(k)$ is updated. In the classical RLS the covariance vanishes to zero with time, losing its capability to keep track of changes in the parameter. In (11) however, the covariance matrix is divided by $\lambda < 1$ at each update. This slows down fading out of the covariance matrix. The exponential convergence of the above scheme is shown in some textbooks and research papers (See e.g. the proof provided in [7] or [16]) for the case of unknown but "constant" invariant case. In general exponential convergence in the constant case implies certain degree of tracking capability in the time varying case [9]. However rigorous mathematical analysis of tracking capabilities of an estimator when the parameters are time-varying is rare in literature and many properties are demonstrated through simulation results. Campi [9] provides rigorous mathematical arguments that if the covariance matrix of the estimator is kept bounded the tracking error will remain bounded. Ljung and Gunnarsson present a survey of algorithms for tracking time-varying systems in [20].

The RLS with forgetting has been widely used in estimation and tracking of time-varying parameters in various fields of engineering. However when excitation of the system is poor this scheme can lead to the covariance "wind-up" problem. During poor excitations old information is continuously forgotten while there is very little new dynamic information coming in. This might lead to the exponential growth of the covariance matrix and as a result the estimator becomes extremely sensitive and therefore susceptible to numerical and computational errors [12]. This problem has been investigated by many researchers in the field and several solutions, mostly ad hoc, have been proposed to avoid covariance "wind-up". The idea of most of these schemes is to limit the growth of covariance matrix for example by introducing an upper bound. A popular scheme is proposed by Fortescue et al. [12] in which a time-varying forgetting factor is used. During low excitations, the forgetting factor is closer to unity to enhance the performance of the estimator. In another approach, Sripada and Fisher [28] propose an on/off method along with a time-varying forgetting factor for improved performance. The concept of resetting the covariance matrix during low excitations has been also investigated in [27]. Both papers provide good discussions about behavior of the system during low excitations. Kulhavy and Zarrop discuss the concept of forgetting from a more general perspective in [18].

One other popular refinement to the RLS with forgetting scheme is the concept of "directional forgetting" for reducing the possibility of the estimator windup when the incoming information is non-uniformly distributed over all parameters. The idea is that if a recursive forgetting method is being used, the information related to non-excited directions will gradually be lost. This results in unlimited growth of some of the elements of the covariance matrix and can lead to large estimation errors. Implementation of the concept of directional forgetting is again ad hoc and is reflected in updating the covariance matrix, $P(k)$. That is, if the incoming information is not uniformly distributed in the parameter space the proposed schemes perform a selective amplification of the covariance matrix. Hagglund [14] and Kulhavy [17] have developed one of the early versions of this algorithm. Bittani et al. discuss the convergence of RLS with directional forgetting in [6]. Cao and Schwartz [10] explain some of the limitations of the earlier directional forgetting scheme and propose an improved directional forgetting approach.

The estimator wind-up can also occur if we are estimating multiple parameters that each (or some) vary with a different rate. This scenario is of particular interest in the problem of mass and grade estimation where mass is constant and grade is time-varying. It will be shown by simulation later in this chapter that the single forgetting algorithm is not able to track parameters with different variation rates. Therefore it is desirable to assign different forgetting factors to different parameters. This problem is somehow similar but not equivalent to the case when excitations are non-uniform in the parameter space. Even when all the modes are uniformly excited, different rate of variations of parameters can cause trouble in estimation. An ad hoc remedy to this problem has been suggested in a few publications and is known as vector-type forgetting [25], [26] or selective forgetting [23]. The idea is again implemented in the update of covariance matrix. Instead of dividing all elements by a single λ, P is scaled by a diagonal matrix of forgetting factors:

$$P(k) = \Lambda^{-1} \left(I - L(k)\phi^T(k) \right) P(k-1)\Lambda^{-1} \tag{12}$$

where

$$\Lambda = diag[\lambda_1, \lambda_2, \ldots, \lambda_n]$$

in an n-parameter estimation and λ_i is the forgetting factor reflecting the rate of the change of i^{th} parameter. We found this method an effective way of keeping track of the parameters that change with different rates. The few examples of application of this scheme, to the best knowledge of the authors, can be found in [34], and [21]. Yoshitani and Hasegawa [34] have used a vector-type forgetting scheme for parameter estimation in control of strip temperature for the heating furnace. For a self-tuning cruise control Oda et al. [21] propose using vector-type forgetting for detecting step changes in the parameters of a transfer function.

Like most other modifications to RLS with forgetting, mathematical proofs for tracking capabilities of the method, to the best knowledge of the authors, do not exist. However a proof for convergence to constant parameter values can be found in [24]. In [24] a general class of RLS with forgetting is formulated and vector type forgetting is also included as a special case. Exponential convergence to constant parameter values is proven for this general class of estimators.

Before employing the vector-type forgetting, and to remedy the problems associated with different rates of variations, the authors had formulated a multiple forgetting method which has similarities to and differences from the above-mentioned schemes. It has shown very good convergence and tracking capabilities in simulation and experiments and the way it is formulated makes an intuitive sense. Since it provides some motivation on the concept of multiple forgetting, we discuss the formulation and the structure of the problem in the next section. The convergence or conditions for convergence of the algorithm remains open for future research.

4.2 A Recursive Least Square Scheme with Multiple Forgetting

When working on the particular mass and grade estimation problem, the authors noticed that the difficulties in RLS with single forgetting stem from the following facts: 1. In the standard method it is assumed that the parameters vary with similar rates. 2. In the formulation of the loss-function defined in (9) and subsequently the resulting recursive scheme, the errors due to all parameters are lumped into a single scalar term. So the algorithm has no way to realize if the error is due to one or more parameters. As a result if there is drift in a single parameter, corrections of the same order will be applied to all parameters which results in over-shoot or undershoot in the estimates. If the drift continues for sometime it might cause poor overall performance of the estimator or even the so-called estimator "wind-up" or "blow-up". It is true that we are fundamentally restricted by the fact that the number of existing equations is less than number of parameters which we are estimating, but in a tracking problem we can use our past estimation results more wisely by introducing some kind of "decomposition" in the error due to different parameters. Therefore, our intention is to conceptually "separate" the error due to each parameter and subsequently apply a suitable forgetting factor for each. Without loss of generality and for more simple demonstration, we shall assume that there are only two parameters to estimate. We define the loss function, V:

$$\begin{aligned} V(\hat{\theta}_1(k), \hat{\theta}_2(k), k) = \;& \tfrac{1}{2}\textstyle\sum_{i=1}^{k} \lambda_1^{k-i} \left(y(i) - \phi_1(i)\hat{\theta}_1(k) - \phi_2(i)\theta_2(i)\right)^2 + \\ & \tfrac{1}{2}\textstyle\sum_{i=1}^{k} \lambda_2^{k-i} \left(y(i) - \phi_1(i)\theta_1(i) - \phi_2(i)\hat{\theta}_2(k)\right)^2 . \end{aligned} \tag{13}$$

With this definition for the loss function the first term on the right hand side of (13) represents only the error of the step k due to first parameter estimate, $\hat{\theta}_1(k)$ and the second term corresponds to the second parameter estimate, $\hat{\theta}_2(k)$. Now each of these errors can be discounted by an exclusive forgetting factor. Notice that $\theta_1(k)$ and $\theta_2(k)$ are unknown. We will later replace them with their estimates, $\hat{\theta}_1(k)$ and $\hat{\theta}_2(k)$. The swapping between the estimated and the actual parameters allows us to formulate the proposed modification to the classical LS with forgetting factors.

Here λ_1 and λ_2 are forgetting factors for first and second parameters respectively. Incorporating multiple forgetting factors provides more degrees of freedom for tuning the estimator, and as a result, parameters with different rates of variation could possibly be tracked more accurately. The optimal estimates are those that minimize the loss function and are obtained as follows:

$$\frac{\partial V}{\partial \hat{\theta}_1(k)} = 0 \Rightarrow \sum_{i=1}^{k} \lambda_1^{k-i} \left(-\phi_1(i)\right) \left(y(i) - \phi_1(i)\hat{\theta}_1(k) - \phi_2(i)\theta_2(i)\right) = 0 \tag{14}$$

Rearranging (14), $\hat{\theta}_1(k)$ is found to be:

$$\hat{\theta}_1(k) = \left(\sum_{i=1}^{k} \lambda_1^{k-i}\phi_1(i)^2\right)^{-1} \left(\sum_{i=1}^{k} \lambda_1^{k-i} \left(y(i) - \phi_2(i)\theta_2(i)\right)\right) \tag{15}$$

Similarly $\hat{\theta}_2(k)$ will be:

$$\hat{\theta}_2(k) = \left(\sum_{i=1}^{k} \lambda_2^{k-i} \phi_2(i)^2 \right)^{-1} \left(\sum_{i=1}^{k} \lambda_2^{k-i} \left(y(i) - \phi_1(i)\theta_1(i) \right) \right) \tag{16}$$

For real time estimation a recursive form is required. Using the analogy that is available between (15), (16) and the classical form (5), the recursive form can be readily deduced:

$$\hat{\theta}_1(k) = \hat{\theta}_1(k-1) + L_1(k) \left(y(k) - \phi_1(k)\hat{\theta}_1(k-1) - \phi_2(k)\theta_2(k) \right) \tag{17}$$

where

$$L_1(k) = P_1(k-1)\phi_1(k) \left(\lambda_1 + \phi_1^T(k) P_1(k-1) \phi_1(k) \right)^{-1}$$
$$P_1(k) = \left(I - L_1(k)\phi_1^T(k) \right) P_1(k-1) \frac{1}{\lambda_1}.$$

and similarly,

$$\hat{\theta}_2(k) = \hat{\theta}_2(k-1) + L_2(k) \left(y(k) - \phi_1(k)\theta_1(k) - \phi_2(k)\hat{\theta}_2(k-1) \right) \tag{18}$$

where

$$L_2(k) = P_2(k-1)\phi_2(k) \left(\lambda_2 + \phi_2^T(k) P_2(k-1) \phi_2(k) \right)^{-1}$$
$$P_2(k) = \left(I - L_2(k)\phi_2^T(k) \right) P_2(k-1) \frac{1}{\lambda_2}.$$

In the two aforementioned equations $\theta_1(k)$, $\theta_2(k)$ are unknown. We propose to replace them with their estimates, $\hat{\theta}_1(k)$ and $\hat{\theta}_2(k)$, as is customary in similar situations, such as the "separation principle" in optimal control [8]. The substitution is also justified when the actual and the estimated values are very close to each other or within the algorithm region of convergence. A convergence proof or conditions for convergence of the algorithm under this assumption, remains open for future research. Upon substitution for $\theta_1(k)$ and $\theta_2(k)$ and rearranging (17) and (18) we obtain:

$$\hat{\theta}_1(k) + L_1(k)\phi_2(k)\hat{\theta}_2(k) = \hat{\theta}_1(k-1) + L_1(k) \left(y(k) - \phi_1(k)\hat{\theta}_1(k-1) \right) \tag{19}$$

$$L_2(k)\phi_1(k)\hat{\theta}_1(k) + \hat{\theta}_2(k) = \hat{\theta}_2(k-1) + L_2(k) \left(y(k) - \phi_2(k)\hat{\theta}_2(k-1) \right) \tag{20}$$

For which the solution is,

$$\begin{bmatrix} \hat{\theta}_1(k) \\ \hat{\theta}_2(k) \end{bmatrix} = \begin{bmatrix} 1 & L_1(k)\phi_2(k) \\ L_2(k)\phi_1(k) & 1 \end{bmatrix}^{-1} \begin{bmatrix} \hat{\theta}_1(k-1) + L_1(k) \left(y(k) - \phi_1(k)\hat{\theta}_1(k-1) \right) \\ \hat{\theta}_2(k-1) + L_2(k) \left(y(k) - \phi_2(k)\hat{\theta}_2(k-1) \right) \end{bmatrix} \tag{21}$$

Using the fact the P_1 and P_2 are always positive it can be proved that the determinant of the matrix

$$\begin{bmatrix} 1 & L_1(k)\phi_2(k) \\ L_2(k)\phi_1(k) & 1 \end{bmatrix}$$

is always nonzero and therefore the inverse always exists. With some more mathematical manipulations, (21) can be written in the form of (6):

$$\hat{\theta}(k) = \hat{\theta}(k-1) + L_{new}(k)\left(y(k) - \phi^T(k)\hat{\theta}(k-1)\right) \tag{22}$$

where $L_{new}(k)$ is defined as follows:

$$L_{new}(k) = \frac{1}{1 + \frac{P_1(k-1)\phi_1(k)^2}{\lambda_1} + \frac{P_2(k-1)\phi_2(k)^2}{\lambda_2}} \begin{bmatrix} \frac{P_1(k-1)\phi_1(k)}{\lambda_1} \\ \frac{P_2(k-1)\phi_2(k)}{\lambda_2} \end{bmatrix} \tag{23}$$

The proposed method incorporates different forgetting factors for each parameter. To this end, it does what the vector-type forgetting method does. Eq. (22) is similar in form to the standard update of (6). However the gains of the standard and the proposed form are different. Specifically the former has a crossterm $P_{12}(k-1)$, while the latter does not. In other words the covariance matrix of the proposed method is diagonal. This will result in update of the two parameters proportional to $P_1(k)$ and $P_2(k)$.

In short, introduction of the loss-function (13) with decomposed errors and different forgetting factors for each have two distinct implications:

1) Parameters are updated with different forgetting factors. That is done by scaling the covariances by different forgettings. This is more or less what is done in the RLS with vector-type forgetting as well. However this approach is based on minimization of a loss-function.

2) It decouples the updating step of the covariance of different parameters. This is different from standard RLS or RLS with vector-type forgetting. It is more similar to the "simplified" algorithms mentioned in second chapter of [3]. The authors believe that when the parameters are independent of each other this makes an intuitive sense.

This scheme did well in both simulation and experiments of mass-grade estimation. The performance is very similar to the RLS with vector forgetting when similar forgetting factors are used. However we observed in some simulations that if the value of the forgetting factors are picked in a way that mismatches real rate of variations of the parameters, RLS with vector forgetting sometimes winds up. In such a situation the estimator was excessively sensitive to noise. On the other hand, the proposed scheme behaved well in this scenario and mismatch between forgetting and true rate of variations did not cause the windup behavior. In other words the proposed algorithm seems to be "forgiving" to the choice of forgetting factors. In the following section we carefully select the forgetting factors of the vector-type forgetting RLS so that the response compares favorably with the decoupled multiple forgetting that we proposed.

4.3 Simulation Analysis of Single and Multiple Forgetting Methods

We first use simulated data to test a recursive scheme for estimation of mass and grade. The simulated data was generated using the vehicle dynamics model given in (2) and by assuming different road grade profiles and feasible mass and parameters for a heavy duty vehicle. A feasible fuelling pattern was chosen. Variation of fuelling is important in exciting all modes of the system and consequently allows better estimation results. Therefore in generating the fuelling command this was taken into account. The engine torque was then calculated based on fuelling rate and engine speed by using the engine torque map. At this stage we assumed that no gear change occurs during the estimation process. In the next sections, we will discuss the issue of gear change and explain how it can be incorporated in experimental estimation. We use a recursive least square scheme

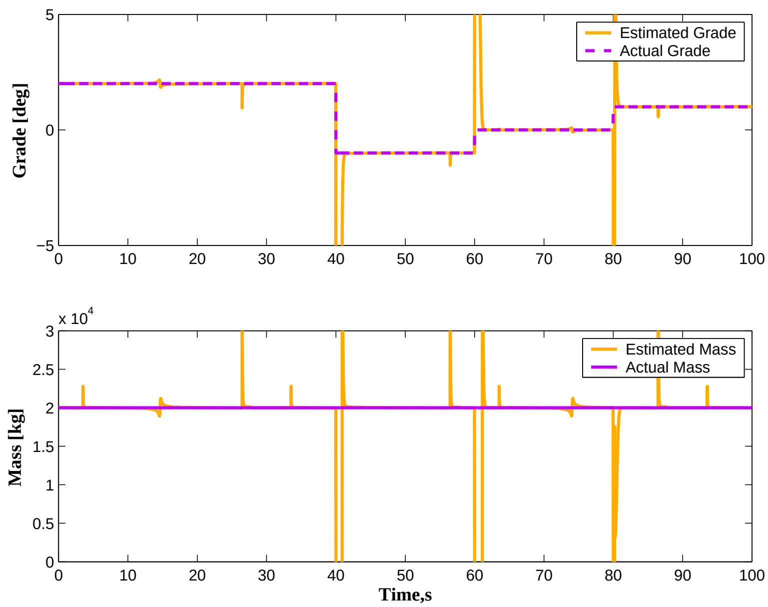

Figure 3: Estimation of mass and grade using RLS with a single forgetting factor of 0.8 when grade is piecewise constant. Sampling rate is 50 Hz. The spikes during steady-state are due to step changes in fuelling rate.

for estimating and tracking the parameters. For initialization, we employ a direct least square on a batch of first few seconds of data. This initializes the estimates and the P matrix. Sampling rate of all signals is 50 Hz.

First we used the RLS with single forgetting for estimation of mass and grade. For the reasons explained in previous sections of this chapter the results were not promising at all. First we considered a constant mass and step changes in grade. The data that we used was clean from any noise. Figure 3 shows the estimation results. We observe big overshoots or undershoots in both mass and grade estimates during step changes in grade or fuelling.

Nevertheless we see a relatively fast convergence back to the real parameter values after the deviations. That is in line with the proofs of convergence of RLS with or without forgetting to constant parameter values. The spikes during the steady-state are due to step changes in fuelling rate which act similar to a disturbance to the system. To this end, despite the local misbehavior we can still get some estimates for both parameters.

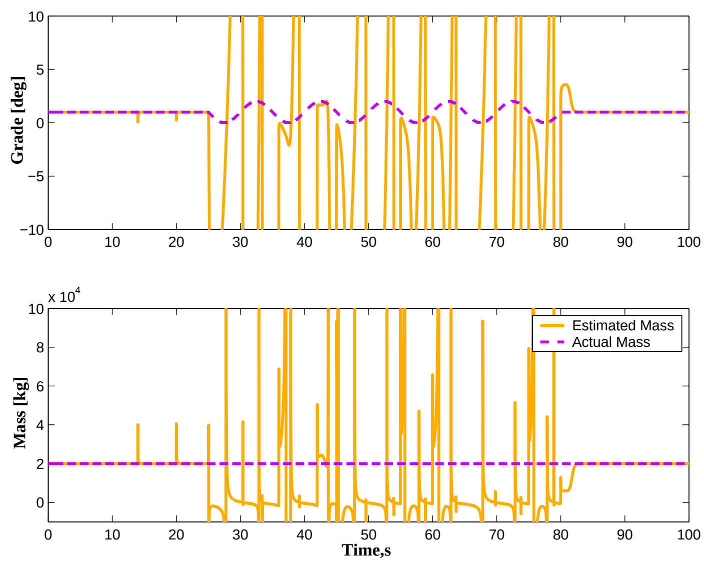

Figure 4: Estimation of mass and grade using RLS with a single forgetting factor of 0.9 when grade variations are sinusoidal. Smaller forgetting factors for grade resulted in worsens the performance. The spikes during steady-state are due to step changes in fuelling rate.

The main difficulty of the approach appears when one of the parameters, here the grade, starts varying continuously (as opposed to staying piecewise constant). The algorithm shows very poor tracking performance in such a scenario.

Figure 4 shows the estimator performance when grade variations are sinusoidal. The well-known phenomenon of estimator "blow-up" or "wind-up" can be seen during grade changes and errors in both mass and grade estimates become very large. The estimates converge back to the real values only when the grade becomes constant. Here a forgetting factor of 0.9 is chosen. We noticed that reducing the forgetting factor will only worsen the problem. When noise is introduced in the data, the performance is sacrificed even more. Increasing the forgetting factor to 1 (classical RLS) will eliminate the big overshoots, but will average all the past data equally. This can result in meaningless estimates for both parameters. As explained in the formulation of the problem, the reason for the poor performance of RLS with single forgetting is that when an error is detected the estimates for both parameters are updated without differentiating between the ones that vary faster and those that do not vary as often or are constant. This causes overshoot/undershoot in

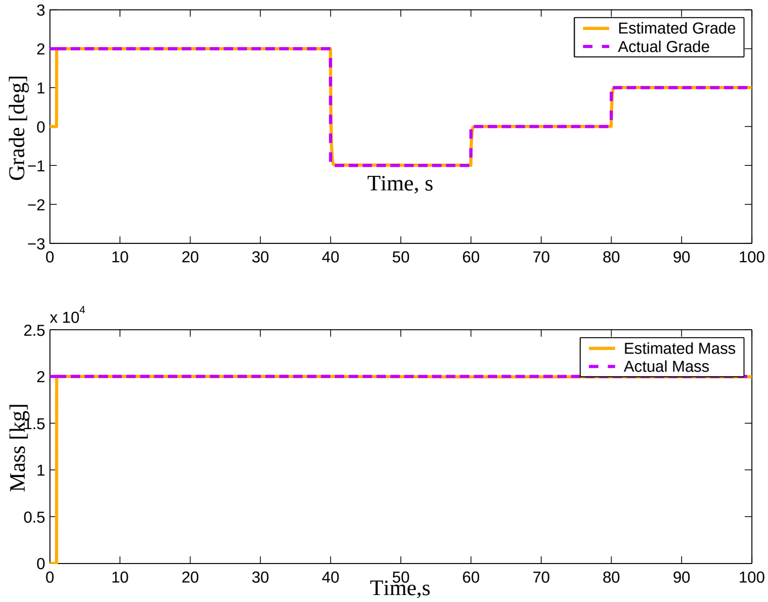

Figure 5: Estimation of mass and grade using RLS with multiple forgetting factors of 0.8 and 1 respectively for grade and mass.

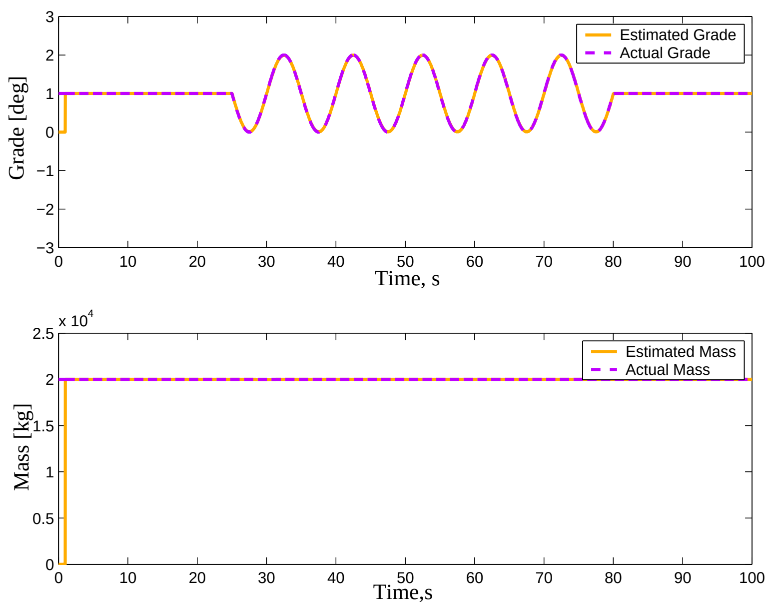

Figure 6: Estimation of mass and grade using RLS with multiple forgetting factors of 0.8 and 1 respectively for grade and mass.

Scenario	Single Forgetting	Multiple Forgetting
Constant grade Constant mass	good estimation	good estimation
Step changes in grade Constant mass	some overshoots in estimates	good estimation
Linear change of grade Constant mass	bad estimation	good estimation
Sinusoidal change of grade constant mass	bad estimation	good estimation
Sinusoidal change of grade Linear variations of mass	bad estimation	estimates with some lag

Table 1: Comparison of the performance of single and multiple forgetting RLS algorithms

the estimates. If one parameter continues drifting, the estimation errors add up to result in what was seen in the previous figures.

We carried out simulations using RLS with multiple forgetting factors and showed that this scheme can resolve the problems encountered with single forgetting. Figure 5 shows the performance of the estimator when grade goes through step-changes. In this example forgetting factors of 0.8 and 1.0 are chosen for grade and mass respectively. Unlike estimation with single forgetting, the estimation is very smooth and the estimates converge much faster during step changes. Also the spikes due to step change in fuelling rate disappear. Because a forgetting factor of 1.0 is chosen for mass, the mass estimates are not as sensitive to changes in grade.

We also tried sinusoidal variations in grade. The results are shown in Figure 4.3. The grade is tracked very well and with very small lag. The rate of change shown for the grade is much faster than the norm on the roads. Even with a much higher speed of variations, the estimator does not ill-behave. In simulation we observed that if the forgetting factors are chosen so that they roughly reflect relative rate of change of parameters, parameter changes are tracked well.

A summary of some other scenarios is shown in Table 1. The results shown in this table are based on numerical data that is not noisy. Simulations with data contaminated by noise show that noise deteriorates the performance of the single forgetting estimation. The multiple forgetting scheme showed much better robustness in presence of noise.

5 Performance of the Estimator with Experimental Data

In the previous sections of this chapter, the estimation problem was formulated, a solution was proposed and it was shown in simulations that it performs well in estimating mass and keeping track of time-varying grade. The demonstration was either in a noise-free environment or when white noise was added to the signals. In a real scenario the situation can become more challenging due to higher level of uncertainties. The signals are potentially delayed and many times the signals are noisy and biased in one direction rather than being only affected by pure white noise. Moreover, the delay or noise level in one signal is normally different from the other signals. Finally, note that what is available from sources like J1939 is normally not the true value of an entity but an estimate of the true value through the vehicle/engine management system. Unmodelled dynamics of the system might result in poor estimation.

The signals in a natural experimental cycle may not always be persistently exciting. As discussed before lack of good excitation results in poor estimates or even cause estimator windup. In our case, if the acceleration is constant and there is no gear change, we are not able to observe enough to determine both mass and grade. In this case the longitudinal dynamics equation represents essentially a single mode, making it literally impossible to estimate the two unknowns. Therefore it is important that in online estimation, rich pieces of data are detected and used for estimation of both parameters. Once a good estimate for mass which is constant is obtained tracking of variations of grade would be possible even during low or constant levels of acceleration.

5.1 Modification for Reducing Signal Noise Effect

Direct implementation of (2) in least square estimation requires differentiation of velocity and engine speed signals. In a noisy environment, differentiation is not very appealing. It will magnify the noise levels to much higher values and the differentiated data may not be useable. In order to circumvent this problem we will first integrate both sides of (2) over time and apply the estimation scheme to the new formulation. Assuming that mass and coefficient of rolling resistance are constant, integration of both sides yields:

$$v(t_k) - v(t_0) = \frac{1}{M}\int_{t_0}^{t_k} (\frac{T_e(t) - J_e\dot{\omega}(t)}{r_g(t)} - F_{fb}(t) - F_{aero}(t))dt - \frac{g}{\cos(\beta_\mu)}\int_{t_0}^{t_k} \sin(\beta + \beta_\mu)dt \tag{24}$$

We can rewrite the above equation in the form of (3),

$$y = \phi^T\theta,\ \phi = [\phi_1, \phi_2]^T,\ \theta = [\theta_1, \theta_2]^T$$

where this time

$$y(k) = v(t_k) - v(t_0)$$

$$\theta = [\theta_1, \theta_2]^T = [\frac{1}{M},\ \frac{\int_{t_0}^{t_k} \sin(\beta + \beta_\mu)dt}{(t_k - t_0)}]^T$$

and

$$\phi_1 = \int_{t_0}^{t_k} (\frac{T_e - J_e\dot{\omega}}{r_g} - F_{fb} - F_{aero})dt,\ \phi_2 = -\frac{(t_k - t_0)g}{\cos(\beta_\mu)}$$

Notice that ϕ_2 is multiplied by $(t_k - t_0)$ and θ_2 is divided by it. This is to keep the unknown parameter θ_2 away from growing fast with time. In this fashion if the grade, β, is constant,

θ_2 will remain constant as well. Employing integration instead of differentiation helped avoid some serious issues related to signal noise.

5.2 Estimation in Normal Cruise: No Gearshift

We first evaluate the estimation scheme with experimental data when the gear is constant. Similar to the approach in simulations we use a batch in the first few seconds of estimation to initialize the estimation scheme. Good initial estimates are obtained only when the chosen batch is rich in excitations. Better estimates can be obtained with a smaller batch when the acceleration has some kind of variation during the batch. The RLS with multiple forgetting was used during the rest of the travel for estimation and tracking.

To reduce the high frequency noise, the torque and velocity signals were passed through a second order butterworth filter before they were used in the estimation. The sampling frequency is 50 Hz, and therefore the Nyquist frequency is 25 Hz. We use the cutoff frequency of 25 Hz for the filter, to ensure that aliasing will not occur. Figure 7 shows

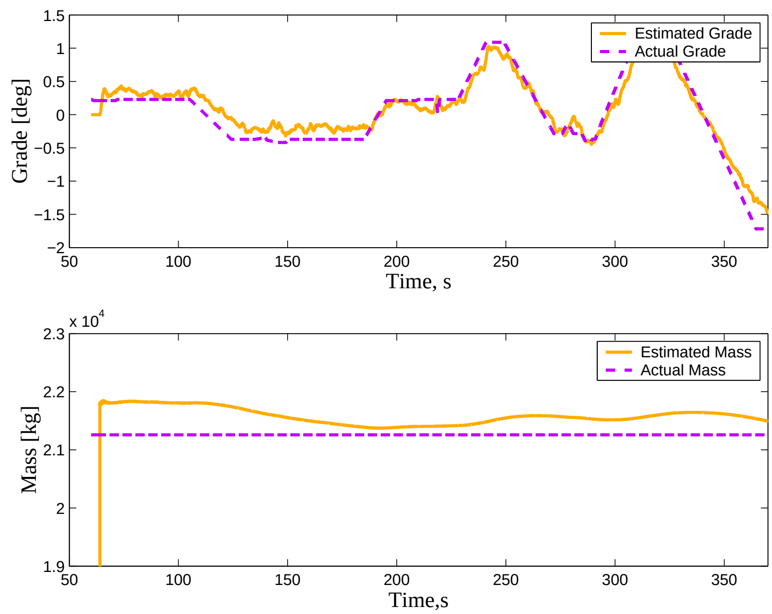

Figure 7: Estimator's performance during normal cruise when the gear is constant. Forgetting factors for mass and grade are 0.95 and 0.4 respectively. RMS error in mass is 350 kg and RMS grade error is 0.2 degrees.

the estimation results for more than five minutes of continuous estimation. The gear was constant throughout this period. The initial four seconds of data was processed in a batch to generate the initial estimates. For the recursive part forgetting factors of 0.95 and 0.4 were chosen for mass and grade respectively. While mass is constant, a slight forgetting acts as a damping effect on the older information and makes the mass estimate a little more responsive to new information. This showed to result in further convergence of mass to its true value. In this estimation the root mean square (RMS) error in mass is 350 kg and the maximum error is 2.8%. During the recursive section the error in mass reduces down to a maximum of 1.7%. The RMS error in grade is 0.2 degrees. It can be seen that grade is estimated well during its variations.

Next we will remedy the estimation problem when gear changes occur.

5.3 Estimation Results During Gearshift

In the longitudinal dynamics Eq. (1) we assume that engine power passes continuously through the driveline to the wheels. This assumption is valid only when the transmission and torque convertor are fully engaged. During a gear change, transmission disengages to shift to the next gear and during this time the flow of power to the wheels is reduced and in the interval of complete disengagement no torque is passed over to the wheels. Moreover the assumption that vehicle speed is proportional to the engine speed by some driveline ratio is not in effect during this transition and the engine speed goes through abrupt changes while the change in vehicle velocity is much smoother. Therefore relying

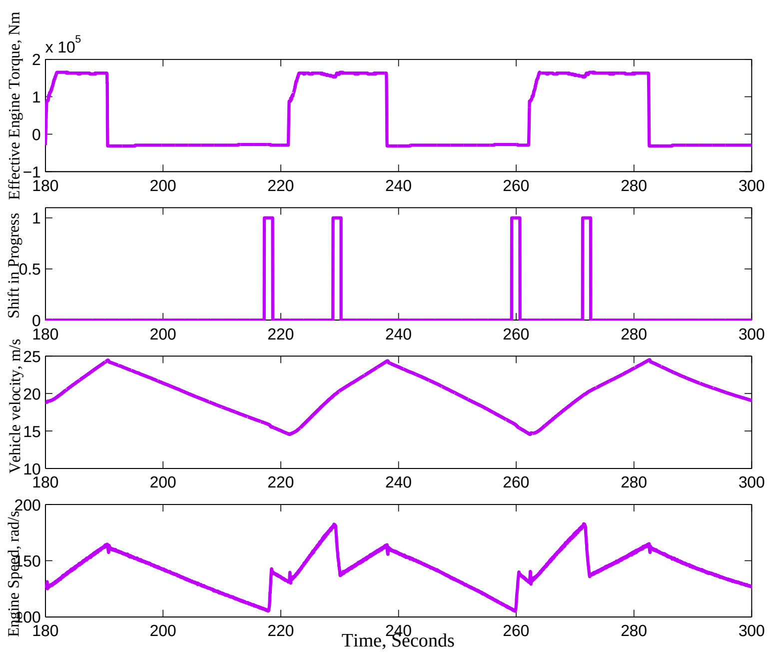

Figure 8: The response during a cycle of pulsing the throttle

on (1) for estimation will result in very big deviations during gearshift. The bigger the deviations are the longer it takes the estimator to converge back to the true parameter values.

Modelling the dynamics during a shift is not simple due to natural discontinuities in the dynamics. Besides the period when the transmission is in control does not take more than two seconds and therefore it is not really necessary to estimate the parameters during this short period. Therefore we decided to turn off the estimator at the onset of a gearshift and turn it back on a second or two after the shift is completed. The estimates during the shift are set equal to the latest available estimates. Also the new estimator gain is set equal to the latest calculated gain. This approach proved to be an effective way of suppressing unwanted estimator overshoots during gear shift. Figure 8 shows the engine

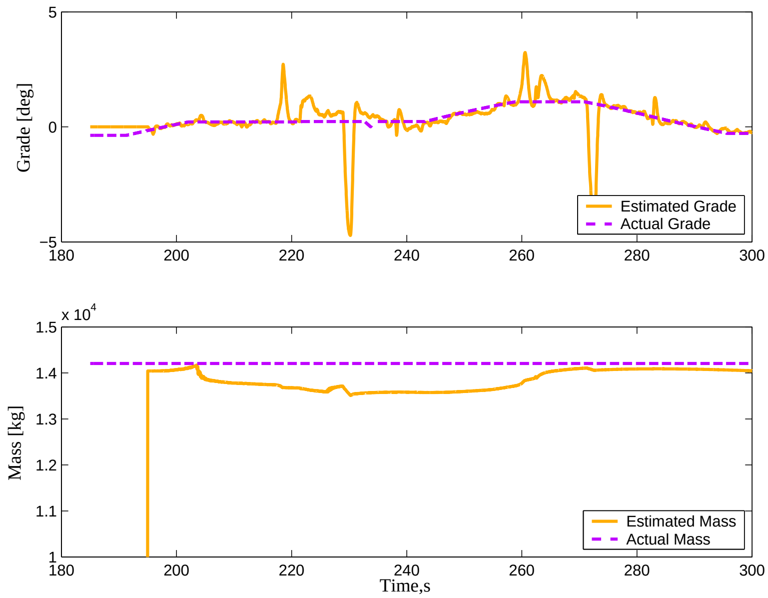

Figure 9: Estimator's performance when it is always on. Forgetting factors for mass and grade are 0.95 and 0.4 respectively. The RMS errors in mass and grade are 420 kg and 0.77 degrees respectively.

torque, shift status, vehicle velocity and engine speed during part of an experiment. We had asked the driver to pulse the throttle off and on and therefore as seen in the torque plot, the torque is either at its maximum or drops down to zero. Also two gear shifts occur during this time window. As mentioned before the variations in velocity are smooth but the engine speed has jump discontinuities both during gear shift and during the throttle on/off. Upon using the estimator with no on/off logic we observed big overshoots in the estimates during both the gearshift and the throttle on/off. The results are shown in Figure 9. The root mean square error in mass is 420 Kilograms and the RMS grade error is 0.77 degrees which is a large error. We then used the estimator with the on/off logic. The results are shown in figure 10. The estimation has improved considerably due to the estimator deactivation during the shifts. The deviations due to throttle pulsation exist as before but the magnitude of these deviations are small and they fade away quickly. In this estimation the root means square error in mass is 310 kilograms and the RMS grade error is 0.24 degrees which are quite improved due to the employed estimator logic.

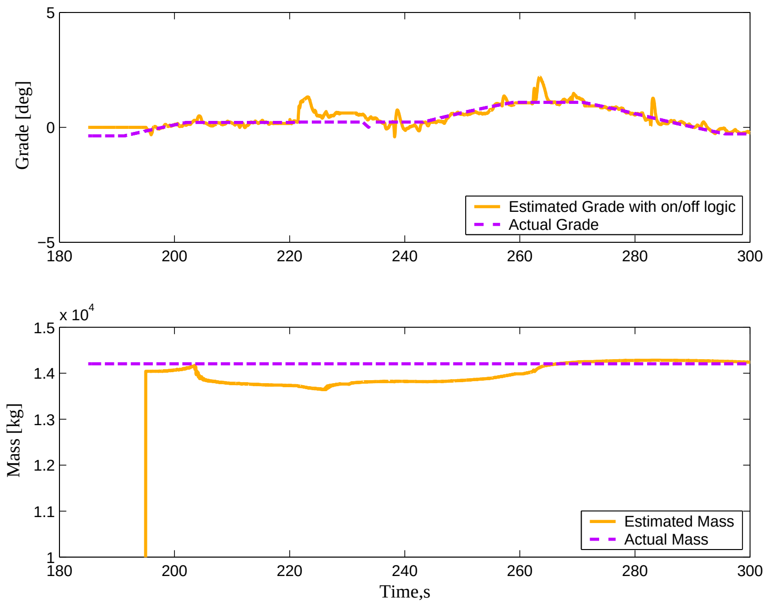

Figure 10: Estimator's performance when it is turned off during shift. Forgetting factors for mass and grade are 0.95 and 0.4 respectively. The RMS errors in mass and grade are 310 kg and 0.24 degrees respectively.

5.4 Sensitivity Analysis

Earlier in this paper the coefficient of rolling resistance and the drag coefficient were calculated based on matching the model outcomes and experimental results. We mentioned that these estimates are rough estimates that meet our needs. We are in general interested to know how much the mass and grade estimation results are sensitive to these parameters. In other words we want to analyze the sensitivity of the estimation scheme with respect to these parameters.

For this analysis we vary the rolling resistance and drag coefficient one at a time and observe the performance of the estimates and based on these results provide a sense on the sensitivity of the system. We perform the analysis with the experimental set of data used in section 5.2 of this paper. Figure 11 shows the sensitivity of the estimates with respect to drag coefficient and rolling resistance. Variations in the coefficient of rolling resistance only affect the grade estimate. That is because the rolling resistance and grade affect the longitudinal dynamics in the same way. In a realistic range, a 50% variation of the coefficient of rolling resistance caused, in the worst case, less than 25% change in the RMS error of grade estimates. The drag coefficient selection influenced both mass and grade estimates. Here 25% change in drag coefficient within a feasible range, cause less than 25% change of error in grade and mass estimates.

In the analysis of this paper the wheel radius was known accurately. However to see how would an incorrect measure of wheel radius affect the estimation results, we carried out sensitivity analysis for different wheel radii. The results are shown in Figure 12. The

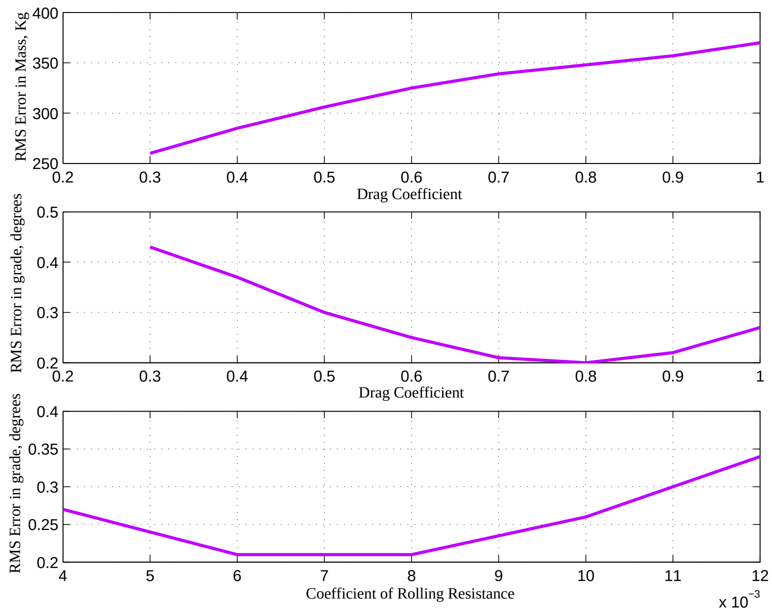

Figure 11: Sensitivity of the estimates with respect to drag coefficient and rolling resistance. Forgetting factors for mass and grade are 0.95 and 0.4 respectively. Nominal mass is 21250 kg.

results show that grade estimates are not very sensitive to errors in wheel radius while sensitivity is roughly 1 for mass estimate. This was expected as wheel radius directly affects the available traction torque for acceleration which in turn directly affects mass estimates. Therefore it is important that accurate value for wheel radius is used to successfully estimate the mass.

6 Conclusions

Simultaneous estimation of vehicle's mass and road grade is a challenging problem. Previous work concentrated on either estimating only one or assumed existence of additional sensors on the vehicle which could be used to estimate one of the unknowns. In this paper a recursive least square scheme with forgetting is proposed for simultaneous online estimation of mass and grade. We show in simulations that a single forgetting factor could not estimate parameters with different rates of variation. Ways to incorporate more than one forgetting factor for estimation of multiple parameters with different rates of variation are discussed and the effectiveness of the algorithm with multiple forgetting in estimating a constant mass and time-varying grade is shown with simulations.

Results of estimation of mass and grade with experimental data are then shown. The data was obtained from experiments that were carried out on Interstate 15 in San Diego in the August of 2002 with an experimental heavy duty vehicle. The experiment setup, the

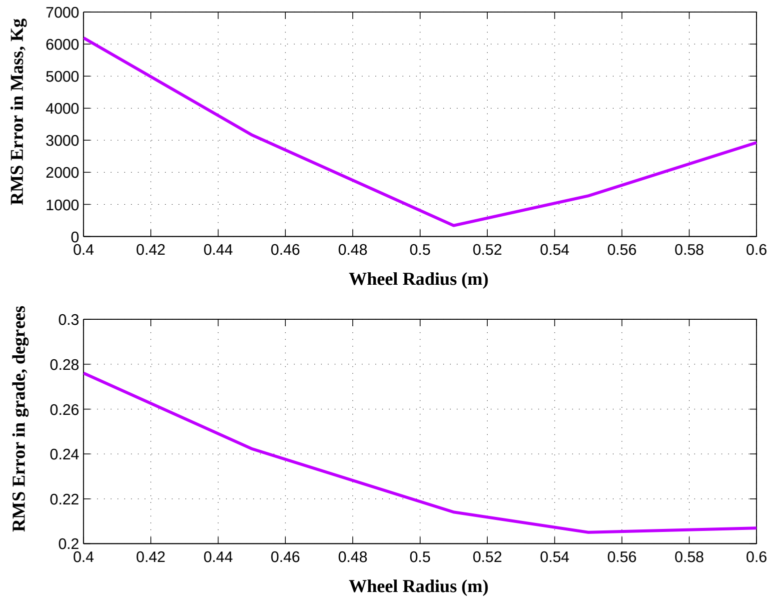

Figure 12: Sensitivity of the estimates with respect to wheel radius. Forgetting factors for mass and grade are 0.95 and 0.4 respectively. Nominal mass is 21250 kg.

measured signals and their source and issues like sampling rate and accuracy are briefly discussed. Using this data we first verify that the vehicle model captures the longitudinal dynamics accurately for most part of travel. The RLS with multiple forgetting which was successful in simulations was tested and proved effective with the experimental data. The real life issues like lack of persistent excitations in certain parts of the run or difficulties of parameter tracking during gear shift are explained and suggestions to bypass these problems are made. Without gear shift and in the presence of persistent excitations mass and grade are estimated with good precision and variations of grade are tracked. When gearshifts takes place, the estimator shows large overshoots and it takes a few seconds for these deviations to damp out. We proposed turning off the estimator during and shortly after a gearshift. The estimation results are improved by this provision. Sensitivity analysis demonstrates that estimation is not overly sensitive to uncertain parameters of the system including drag coefficient and rolling resistance.

In its present form, the proposed scheme can be employed in a real-time application with caution, since its convergence and region of convergence has not been shown. Care should be exercised in choosing the batch initialization procedure and in ensuring persistent excitation. There is room for including some more logical checks and routines that can make the algorithm more robust to a variety of operating situations. Inclusion of a logic to detect areas of high or low excitations is one example which can save the estimator from a potential windup. With the added robustness the proposed scheme can be used alone or along with other sensor or model based schemes for online estimation. We are planning to test this scheme in conjunction with the longitudinal control module and analyze potential improvements to the heavy duty vehicle automation.

Acknowledgement

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References

[1] *Recommended Practice for a Serial Control and Communications Vehicle Network.* Society of Automotive Engineers, 1998.

[2] *Surface Vehicle Recommended Practice; Vehicle Application Layer.* Society of Automotive Engineers, 1998.

[3] A. Astrom and B. Wittenmark. *Adaptive Control.* Addison Wesley, second edition, 1994.

[4] H.S. Bae and J. Gerdes. Parameter estimation and command modification for longitudinal control of heavy vehicles. *Proc. of International Symposium on Advanced Vehicle Control*, 2000.

[5] H.S. Bae, J. Ryu, and J. Gerdes. Road grade and vehicle parameter estimation for longitudinal control using GPS. *Proc.of IEEE Conference on Intelligent Transportation Systems*, 2001.

[6] S. Bittani, P. Bolzern, and M. Campi. Convergence and exponential convergence of identification algorithms with directional forgetting factor. *Automatica*, 26, 5:929–932, 1990.

[7] S. Bittani, P. Bolzern, M. Campi, and E. Coletti. Deterministic convergence analysis of RLS estimators with different forgetting factors. *Proceedings of the 27th Conference on Decision and Control*, pages 1530–1531, 1988.

[8] A.E. Bryson and Y.C. Ho. *Applied Optimal Control: Optimization, Estimation, and Control.* Hemisphere Publishing, New York, 1975.

[9] Marco Campi. Performance of RLS identification algorithms with forgetting factor: A Phi-mixing approach. *Journal of Mathematical Systems, Estimation and Control*, 4, 3:1–25, 1994.

[10] Liyu Cao and Howard M. Schwartz. A novel recursive algorithm for directional forgetting. *Proceedings of the American Control Conference*, pages 1334–1338, 1999.

[11] M. Druzhinina, A. Stefanopoulou, and L. Moklegaard. Adaptive continuously variable compression braking control for heavy-duty vehicles. *ASME Dynamic Systems, Measurement and Control*, 124:406–414, 2002.

[12] T.R. Fortescue, L.S. Kershenbaum, and B.E. Ydstie. Implementation of self-tuning regulators with variable forgetting factors. *Automatica*, 17, 6:831–835, 1981.

[13] Thomas A. Genise. Control method system including determination of an updated value indicative of gross combination weight of vehicles. *US Patent No 5,490,063*, 1994.

[14] T. Hagglund. Recursive estimation of slowly time-varying parameters. *Proceedings of IFAC*, pages 1137–1142, 1985.

[15] P. Ioannou and Z. Xu. Throttle and brake control systems for automatic vehicle following. *PATH Research Report UCB-ITS-PRR-94-10*, 1994.

[16] C.R. Johnson. *Lectures on Adaptive Parameter Estimation*. Prentice Hall, 1988.

[17] R. Kulhavy. Restricted exponential forgetting in real-time identification. *Proceedings of IFAC*, pages 1143–1148, 1985.

[18] R. Kulhavy and M.B. Zarrop. On a general concept of forgetting. *International Journal of Control*, 58, 4:905–924, 1993.

[19] M.K. Liubakka, D.S. Rhode, J.R. Winkelman, and P.V. Kokotovic. Adaptive automotive speed control. *IEEE Transactions on Automatic Control*, 38, 7:146–156, 1993.

[20] Lennart Ljung and Svante Gunnarsoon. Adaptation, tracking and system identification - a survey. *Automatica*, 26,1:7–21, 1990.

[21] K. Oda, H. Takeuchi, M. Tsujii, and M. Ohba. Practical estimator for self-tuning automotive cruise control. *Proceedings of the American Control Conference*, pages 2066–2071, 1991.

[22] H. Ohnishi, J. Ishii, M. Kayano, and H. Katayama. A study on road slope estimation for automatic transmission control. *JSAE Review*, 21:322–327, 2000.

[23] J.E. Parkum, N.K. Poulsen, and J. Holst. Selective forgetting in adaptive procedures. *Proceedings of the 11th Triennial World Congress of the IFAC*, 2:137–142, 1990.

[24] J.E. Parkum, N.K. Poulsen, and J. Holst. Recursive forgetting algorithms. *International Journal of Control*, 55, 1:109–128, 1992.

[25] Steiner Saelid, Olav Egeland, and Bjarne Foss. A solution to the blow-up problem in adaptive controllers. *Modeling, Identification and Control*, 6, 1:36–39, 1985.

[26] Steiner Saelid and Bjarne Foss. Adaptive controllers with a vector variable forgetting factor. *Proceedings of the 22nd IEEE Conference on Decision and Control*, pages 1488–1494, 1983.

[27] Mario E. Salgado, Graham C. Goodwin, and H. Middleton Richard. Modified least squares algorithm incorporating exponential resetting and forgetting. *International Journal of Control*, 47, 2:477–491, 1988.

[28] N. Rao Sripada and D. Grant Fisher. Improved least square identification. *International Journal of Control*, 46, 6:1889–1913, 1987.

[29] J.Y. Wong. *Theory of ground Vehicles*. John Wiley and Sons, third edition, 2001.

[30] M. Wuertenberger, S. Germann, , and R. Isermann. Modelling and parameter estimation of nonlinear vehicle dynamics. *Proceedings of ASME Dynamical Systems and Control Division*, 44, 1992.

[31] D. Yanakiev, J. Eyre, and I. Kanellakopoulos. Longitudinal control of HDV's: Experimental evaluation. Technical Report MOU 293, PATH, 1998.

[32] D. Yanakiev and I. Kanellakopoulos. Speed tracking and vehicle follower control design for heavy-duty vehicles. *Vehicle System Dynamics*, 25:251–276, 1996.

[33] D. Yanakiev and I. Kanellakopoulos. Longitudinal control of automated CHV's with significant actuator delays. *Proceedings of CDC*, 25:251–276, 1997.

[34] Naoharu Yoshitani and Akihiko Hasegawa. Model-based control of strip temperature for the heating furnace in continuous annealing. *IEEE Transactions on Control Systems Technology*, 6, 2:146–156, 1998.